The Predictive Performance of Asymmetric Normal Mixture GARCH in Risk Management: Evidence from Turkey

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Abstract

The purpose of this study is to test predictive performance of Asymmetric Normal Mixture Garch (NMAGARCH) and other Garch models based on Kupiec and Christoffersen tests for Turkish equity market. The empirical results show that the NMAGARCH perform better based on %99 CI out-of-sample forecasting Christoffersen test where Garch with normal and student-t distribution perform better based on %95 Cl out-of-sample forecasting Christoffersen test and Kupiec test. These results show that none of the model including NMAGARCH outperforms other models in all cases as trading position or confidence intervals and these results shows that volatility model should be chosen according to confidence interval and trading positions. Besides, NMAGARCH increases predictive performance for higher confidence internal as Basel requires.

Key Words: *Garch, Asymmetric Normal Mixture Garch, Kupiec Test, Christoffersen Test, Emerging markets*

JEL Codes: C52, C32, G0

Özet

Bu çalışmanın amacı, Türk hisse senedi piyasası için Asimetrik Normal Karma Garch (NMAGARCH) ve diğer garch modellerinin öngörü performansını Kupiec ve Christoffersen geriye dönük testleri ile test etmektir. Ampirik bulgular %99 güven aralığı için örneklem dışı Christoffersen testine göre NMAGARCH modelinin, %95 güven aralığı için örneklem dışı Christoffersen ve Kupiec testlerine göre normal ve student-t dağılımlı Garch modelinin diğer modellerden daha iyi sonuç verdiğini göstermektedir. Bu sonuçlar, NMAGARCH modeli de dâhil olmak üzere hiçbir modelin diğer modellere göre tüm posizyon ve güven aralıklarında daha iyi sonuç vermediğini göstermektedir ve bu bulgu volatilite modelinin ticaret posizyonu ve güven aralığına göre seçilmesi gerektiğini göstermektedir. Ayrıca, NMAGARCH modeli Basel'ında gerektirdiği şekilde yüksek güven aralığında öngörü performansını arttırmaktadır.

Anahtar Kelimeler: *Garch, Asimetrik Normal Karma Garch, Kupiec Testi, Christoffersen Testi, Gelişmekte Olan piyasalar*

JEL Sınıflaması: C52, C32, G0

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1. Introduction

Modeling return volatility of the financial instruments is a crucial task for risk management, trading and hedging strategies. Especially in the developing markets in which non-linear behaviors in stock returns and asymmetries in the return volatilities occur due to dynamic and chaotic financial environment, advanced financial modeling techniques are required for accurate and correct estimation of return volatility.

In emerging markets, because of portfolio investments of hedge funds, low market volume and unstable political and economic conditions, the volatility in the returns of financial variables are relatively higher and shows an asymmetric character in that it increases in case of emergence of negative information. What is more, high volatility in the form of shocks causes regime switches, which are not easy to be estimated and modeled with static econometric models.

In the finance literature, among many volatility models, the most successful models are seen as the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models by Bollerslev (1986), who generalizes the seminal idea on ARCH by Engle (1982), and their numerous generalizations that add asymmetries, long memory, or structural breaks. GARCH models are popular due to their ability to capture many of the typical stylized facts of financial time series, such as time-varying volatility, persistence and volatility clustering. Andersen and Bollerslev (1998) find that GARCH models do really provide good volatility forecasts, in particular when a good proxy for the latent volatility, such as the realized volatility, is adopted.

In this paper, five main GARCH models are used to estimate the stock market volatility. In addition, each model is applied on the time series with different normality assumptions, mainly normal distribution, Student's t distribution and skewed Student's t distribution. In recent research, asymmetric normal mixture GARCH models have been used in volatility modeling. Research by Alexander and Lazar (2003, 2005, 2006) uses GARCH(1,1) models with normal mixture conditional densities having flexible individual variance processes and time-varying conditional higher moments.

The importance of using (asymmetric) normal mixture GARCH process lies in the fact that it can captures tails in the financial time series more properly. That is very important for modeling return volatility in the emerging financial markets where asymmetric high volatility observed during financial shocks. The emerging markets are open to internal or external shocks observed due to hot money movements, low trade volume, thin trading and instability. Markov regime switching models are used to capture the effects of the sudden shocks in the emerging markets. The normal mixture GARCH models are similar to Markov switching models and easier for use as it will be explained in the methodology part. This paper tries to estimate the return volatility in the Istanbul Stock Exchange by using five GARCH models including the normal mixture GARCH models with tree different normality distributions. The aim of this research is to examine if the normal mixture GARCH models produce more accurate results and are able to capture shocks as long memory processes.

The paper is constructed as follows. In the next part, a literature review on the predictive performance in return volatility in financial markets is presented. The test results in the literature with different markets and sample periods are compared. In the third part, the methodologies of the GARCH models and different normality distributions are introduced.

The importance is given on the methodology of asymmetric normal mixture GARCH model introduced by Alexander and Lazar (2003, 2005). After presenting the descriptive statistics of the data, empirical tests and Kupiec and Christoffersen back-tests are implemented. The predictive performances of the fifteen GARCH models in-sample and out-of-sample forecasting results are compared. The paper ends with suggestions for risk management and trading functions for their Value-at-Risk calculations and future financial research conducted in the transitory economics.

2. Literature Review

Early empirical evidence has shown that a high ARCH order should be used to capture the dynamics of the conditional variance. The Generalized ARCH (GARCH) process constructed by Bollerslev (1986) solves the problem in the ARCH model. The GARCH model is based on an infinite ARCH specification and reduces the number of estimated parameters by imposing non-linear restrictions on them.

The GARCH models are extended under different motivation and assumption by researchers. The alternative models are EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan, and Runkle, 1993), APARCH (Ding, Granger, and Engle, 1993), IGARCH (Engle and Bollerslev, 1986), FIGARCH (Baillie, Bollerslev, and Mikkelsen, 1996 and Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGRACH (Davidson, 2001). Ackert and Racine (1999), Darrat and Benkato (2003) and Puttonen (1995) use different GARCH models with different markets and time periods and conclude that the GARCH models are successful to model the volatility in the stock returns. As a long memory process, the normal mixture GARCH model captures shocks effects in the time series is used by Alexander and Lazar (2005, 2006).

The GARCH models are used with different assumptions on normality distributions. Bollerslev and Wooldridge (1992) shows that under the normality assumption, the quasi maximum likelihood estimator is consistent if the conditional mean and the conditional variance are correctly specified. This estimator is, however, inefficient with the degree of inefficiency increasing with the degree of departure from normality. Since the issue of fat-tails is crucial in empirical finance, using a more appropriate distribution might reduce the excess kurtosis displayed by the residuals of conditional heteroscedasticity models. Palm (1996), Pagan (1996) and Bollerslev, Chou, and Kroner (1992) use fat-tailed distributions in the literature. Bollerslev (1987), Hsieh (1989), Baillie and Bollerslev (1989) and Palm and Vlaar (1997) show that these distributions perform better in capturing the higher observed kurtosis.

The importance of skewness is explained in many researches. In a recent study, Christoffersen and Jacobs (2004) show that a simple asymmetric GARCH, that captures the leverage effect, performs best of all GARCH model considered. Bekaert and Wu (2000) and Wu (2001) display the fact that the 'leverage effect' in stocks determines a strong negative correlation between returns and volatility, which is the most important reason for skewness in stock returns. Christoffersen, Heston and Jacobs (2004), Bates (1991) focus on the connection between time-variability in the physical conditional skewness and the empirical characteristics of option implied volatility skews.

The difference between the physical and risk neutral skews is among the recent issues in financial research. Bates (2003) states that the difference between the risk-neutral and observed distributions cannot be explained unless the existence of a time-varying volatility risk premium is considered. Bates (2003) conducts the research based on real-world models with a single volatility component. However, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2005) show that GARCH models with time-varying volatility provide a better fit to the physical conditional densities than GARCH specifications with only one volatility state. The conditional higher moments endogenously determined are timevarying in those models. Therefore, their implied volatility skews exhibit the features of risk neutral index skews.

Non-normalities in conditional and unconditional returns is higher than that can be captured by GARCH(1,1) models with normally distributed errors. Bollerslev (1987) constructs GARCH(1,1) model with Student-t distribution. Fernandez and Steel (1998) extend the model to the skewed *t*-distribution. These *t*-GARCH models have no time-variation in the conditional higher moments. On the other hand, Haas, Mittnik and Paolella (2004) and Alexander and Lazar (2006) in their recent researches conduct GARCH(1,1) models with normal mixture conditional densities. The normal mixture GARCH models are flexible in individual variance processes and have time-varying conditional higher moments. Alexander and Lazar (2006) show that if the model has more than two variance components, biases in parameter estimates are likely to result, and the estimated conditional skewness and excess kurtosis can be unstable over time. For modelling major exchange rate time-series, they find that the mixture of two GARCH(1,1) components models outperform both symmetric and asymmetric *t*-GARCH models and normal mixture GARCH(1,1) models with more than two components.

For stock market returns volatility, there are certain discrete time-varying models in the literature based on asymmetric GARCH models Engle and Ng (1993), Glosten, Jagahannathan, and Runkle (1993) Nelson (1991) show that the models capture only one source of skewness, namely, the leverage effect. Additional structure is needed to capture the empirical observations about the nature of skewness in the risk-neutral equity index skew. This paper deals with the problem by using asymmetric normal mixture GARCH model with reality check.

3. Methodology

Reliable forecasting of return volatility in the financial markets is crucial for trading, risk management and derivative pricing. Return volatility is affected by time dependent information flows resulting in pronounced temporal volatility clustering. Therefore, financial time series should be parameterized with Autoregressive Conditional Heteroskedastic (ARCH) models modeling a time-turn varying conditional variance as a linear function of past squared residuals and of its past values. In other words, ARCH models are used to forecast conditional variances in that the variance of the dependent variable is modeled as a function of past values of the dependent variable or exogenous variables. ARCH models are constructed by Engle (1982) and generalized as GARCH (Generalized ARCH) by Bollerslev (1986) and Taylor (1986).

Different GARCH models are used to estimate the return volatility of financial instruments. EGARCH (Nelson, 1991), GJR (Glosten, Jagannathan and Runkle; 1993), APARCH (Ding, Granger and Engle; 1993), IGARCH (Engle and Bollerslev; 1986), FIGARCH (Chung, 1999), FIEGARCH (Bollerslev and Mikkelsen, 1996), FIAPARCH (Tse, 1998) and HYGARCH (Davidson, 2001) are the most known extensions and/or revisions of the ARCH model. The researches show that GARCH models can provide good in-sample parameter estimates and, when the appropriate volatility measure is used, reliable out-of-sample volatility forecasts. Recently the asymmetric normal mixture GARCH model has been used to capture asymmetric volatility in the returns. This paper tests the predictive performance of different GARCH models with normal, Student's t and skewed Student's t distributions of the error terms. Following fifteen models are constructed and compared for estimating return volatility in the Istanbul Stock Exchange.

i) GARCH with normally distributed errors
ii) GARCH with symmetric Student's t distributed errors
iii) GARCH with skewed Student's t distributed errors
iv) GRJ with normally distributed errors
v) GRJ with symmetric Student's t distributed errors
vi) GRJ with skewed Student's t distributed errors
vii) FIGARCH with normally distributed errors
viii) FIGARCH with symmetric Student's t distributed errors
ix) FIGARCH with skewed Student's t distributed errors
x) HYGARCH with normally distributed errors
xi) HYGARCH with symmetric Student's t distributed errors
xii) HYGARCH with skewed Student's t distributed errors
xiii) NM-AGARCH with normally distributed errors
xiv) NM-AGARCH with symmetric Student's t distributed errors
xv) NM-AGARCH with skewed Student's t distributed errors

In a static linear model ($y_i = \alpha + \beta\chi_i + \varepsilon_i$), the error term ($\varepsilon_i$) is accepted as a random variable with normal distribution and constant variance denoted in the Eq. 1.

$$E(\varepsilon_i - 0)^2 = E(\varepsilon_i)^2 = \sigma_\varepsilon^2 \tag{1}$$

Engle (1982) constructs Autoregressive Conditional Heteroscedasticity (ARCH) model to explicit the time-varying variance.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_i^2 = \omega + \alpha(L)\varepsilon_t^2 \tag{2}$$

In the Eq. 2, σ_i is the conditional variance of ε_t and varies on time. The model has restriction that the sum of α_i>0 and α should be 1. In order to reach for estimations with negative variance Bollerslev (1986) constructs Generalized ARCH (GARCH). The GARCH Model includes the effects of both the linear variance and conditional variance of the past.

$$\sigma_t^2 = \alpha_0 + \alpha_1 \sum_{t=2}^{n} \varepsilon_{t-1}^2 + \beta_1 \sum_{t=2}^{n} \sigma_{t-1}^2 \tag{3}$$

The volatility in the returns increases more than the expected with the negative information if there is asymmetry in the time series. The first GARCH model capturing the asymmetry in the volatility is Exponential GARCH constructed by Nelson (1991).

$$In(\sigma_t) = \delta + (1 + \alpha_1 L) f(u_{t-1} / \sigma_{t-1}^{1/2}) + \beta_1 In\sigma_{t-1} \tag{4}$$

$$f(u_{t-1}/\sigma_{t-1}^{1/2}) = \theta u_{t-1} + \gamma(|u_{t-1}/\sigma_{t-1}^{1/2}| - E|u_{t-1}/\sigma_{t-1}^{1/2}|) \tag{5}$$

In the model, the parameters are positive since the logarithmic values of the conditional variance are employed. The Eq. 5 adds the asymmetric characteristic in the model. While "θ" determines the sign of the error term affecting the conditional variance "γ"states the size effect. If there is asymmetry in the time series, θ should be less than zero.

Gloslen, Jagannathan and Runkle (1993), and Zakoian(1994) state that asymmetry in the return volatility can be modeled by adding a dummy variable into GARCH model. GJR (Threshold GARCH) model is shown on Eq. 6.

$$\sigma_t = \alpha_0 + \alpha_1 u_{t-1}^2 + \gamma 1 u_{t-1}^2 I_{t-1} + \beta_1 \sigma_{t-1} \tag{6}$$

In the model, if u_{t-1} higher than zero, I_{t-1} is equal to 1, otherwise, equal to zero. ARCH parameters in the conditional variance vary between $\alpha_1+\gamma 1$ and α_1 in accordance with the sign of the error term. The positive news are affected on the α_1 while the negative news do α_1 and $\gamma 1$. If $\gamma 1$ is higher than 1, it is accepted that there is asymmetry effect while $\gamma 1$ is equal to zero, on the other hand, the news impact curve is symmetric.

ARCH, GARCH and asymmetric GARCH models do not take into consideration the stationary of the conditional variance in the error terms. In the GARCH (1,1) model, if $\alpha_1+\beta_1$<1; u_t is static. The stationary of the conditional variance depends also Alpha and Beta parameters. In the GARCH(p,q) model, if $\alpha_{1,....,p}+\beta_{,....,q1}$<1, in case of a shock, its effect changes the conditional variance in time known as decay factor. When $\alpha_{1,....,p}+\beta_{,....,q1}$=1, the conditional variance behaves like a unit root process and enables the shock effect to change the conditional variance. Therefore, GARCH (p,q) model has the restriction of $\alpha_{1,....,p}+\beta_{,....,q1}$<1 (Harris ve Sollis, 2003).

In time series with high frequency, the sum of the Alpha and Beta parameters for the conditional variance estimated by GARCH (p,q) model is near or equal to 1 meaning that the volatility effects of the last observations in dataset increase. The same situation is valid for mean equation, as well. When sum of all AR and MA parameters is equal to 1, ARIMA process is expected (Laurent and Peters, 2002). The GARCH (p,q) process can be modeled as an ARMA process and written as on the Eq. 7 by using lag operator.

$$[1-\alpha(L)-\beta(L)]\varepsilon_t^2 = \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2) \tag{7}$$

The [1-α (L)-β (L)] function has a unit root, the sum of Alpha and Beta parameters is 1 and gives Integrated GARCH model of Engle and Bollerslev (1986). IGARCH model is denoted in the Eq. 8 (Laurent and Peters 2001).

$$\phi(L)(1-L)\varepsilon_t^2 = \quad \omega + [1-\beta(L)](\varepsilon_t^2 - \sigma_t^2) \tag{8}$$

When the IGARCH process is modeled as a conditional variance of the squared error terms, it can be written in GARCH formulation as in Eq. 9.

$$\sigma_t^2 = \frac{\omega}{1-\beta(L)} + \{1-\phi(L)(1-L)[1-\beta(L)]^{-1}\varepsilon_{t-1}^2 \quad (9)$$

In the time series, if the fractional difference of y_t has a static process, y_t is in the fractional integration. In the $(1-L)^d = y_t = \varepsilon_t$ equation, if d equals to 0, y_t is static and its autocorrelations are zero. If d is 1, on the other hand, y_t has unit root with zero frequency. In case of $0<d<1$, the autocorrelations of y_t slowly reaches into zero. For that reason, the fractionally integrated models are seen as the models including long memory (Harris and Sollis, 2003). The models with long memory requires in case of high volatility and shocks.

Baillie, Bollerslev and Mikkelsen (1996) constructed Fractionally Integrated GARCH (FIGARCH) model by replacing the lag operator with $(1-L)^d$ in the IGARCH model. FIGARCH-BBM is represented in the Eq. 10.

$$\phi(L)(1-L)^d(\varepsilon_t^2-\sigma_t^2)=[1-\beta(L)](\varepsilon_t^2-\sigma_t^2)\varepsilon_t^2 \quad (10)$$

The conditional variance in the FIGARCH (BBM) model is caculated by Eq. 11 where $\omega^*=[1-\beta(L)]^{-1}$, $\lambda(L)=\{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2$, $0<d<1$, and $\sigma_t^2=\omega^*+\lambda(L)$

$$\sigma_t^2=\omega[1-\beta(L)]^{-1}+\{1-[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}\varepsilon_t^2 \quad (11)$$

Chung (1999) modifies the FIGARCH (BBM) model as it is in the Eq. 12 since ω has theoretical problem and difficulties in the modelling in the practice.

$$\sigma_t^2=\sigma_{t-1}^2+\{[1-\beta(L)]^{-1}\phi(L)(1-L)^d\}(\varepsilon_t^2-\sigma_{t-1}^2) \quad (12)$$

In this article, FIGARCH model suggested by Chung (1999) is tested.

Anther integrated model developed by Davidson (2002) as a special version of FIGARCH is Hyperbolic GARCH. Davidson (2002) uses near epoch dependency in order to reach long-term memory (Saltoglu, 2003). HYGARCH model can be written in the Eq. 13 (Laurent and Peters, 2002).

$$\sigma_t^2=\omega[1-\beta(L)]^{-1}+\{1-[1-\beta(L)]^{-1}\phi(L)\{1+\alpha[(1-L)^d]\} \quad (13)$$

Recently, normal mixture GARCH (NM-GARCH) models have been started to use in detecting the shocks and long-term memory in the returns of the financial instruments. According to Alexander and Lazar (2005), NM-GARCH model can be seen as the Markov switching GARCH model in a restricted form where the transition probabilities are independent of the past state. They argue that the NM-GARCH models are easier to estimate than the Markov switching model constructed by Hamilton and Susmel (1994). What is more, in the NM-GARCH models, the individual variances are only tied with each other through their dependence on the error term.

The methodologies of the NM GARCH models are constructed and formulized by Alexander and Lazar (2005). We follow Alexander and Lazar (2005) in presenting the models[*].

[*] We present our thanks to Emeze Lazar for her help in supplying her original article and suggestions.

The asymmetric normal mixture GARCH model has one equation for the mean and K conditional variance components representing different market conditions. The error term has a conditional normal mixture density with zero mean as a weighted average of K normal density functions with different means and variances.

$$\varepsilon_t / I_{t-1} \sim NM(p_1,\ldots\ldots, p_K, \mu_1,\ldots\ldots, \mu_K, \sigma_{1t}^2,\ldots\ldots, \sigma_{Kt}^2), \qquad \sum_{i=1}^{K} p_i = 1, \qquad \sum_{i=1}^{K} p_i \mu_i = 1 \tag{14}$$

From the Eq. 14, the conditional density of the error term is derived as

$$\eta(\varepsilon_t) = \sum_{i=1}^{K} p_i \varphi_i \tag{15}$$

where φ is normal density functions with different constant means μi and different time varying variances σ_{it}^2 for i = 1,…, K.

In the model, it is assumed that K variances follow normal mixture GARCH processes. The NM-GARCH is represented in the Eq. 16.

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2 \qquad \text{for } i\text{=1,....,K} \tag{16}$$

NM-AGARCH based on the Engle and Ng, (1993) model is in Eq. 17.

$$\sigma_{it}^2 = \alpha_0 + \alpha_i (\varepsilon_{t-1}^2 - \lambda_i)^2 + \beta_i \sigma_{it-1}^2 \qquad \text{for } i\text{=1,....,K} \tag{17}$$

NM-GJR GARCH based on Glosten *et al*, (1993) is given by Eq. 18.

$$\sigma_{it}^2 = \alpha_0 + \alpha_i \varepsilon_{t-1}^2 + -\lambda_i d_{t-1}^- \varepsilon_{t-1}^2 + \beta_i \sigma_{it-1}^2 \quad \text{for i=1,....,K;} \tag{18}$$

where d_t^-=1 if ε_t <0, and 0 otherwise.

For both models, the overall conditional variance is

$$\sigma_t^2 = \sum_{i=1}^{K} p_i \sigma_{it}^2 + \sum_{i=1}^{K} p_i \mu_i^2 \tag{19}$$

When K is bigger than 1, the existence of second, third and fourth moments are assured by imposing less stringent conditions than in the single component in which K is equal to 1.
For asymmetric NM-GARCH models, the conditions for the non-negativity of variance and the finiteness of the third moment are represented in the Eq. 20.

$$0 < p_i < 1, \quad i\text{=1,....,}K\text{-1}, \qquad \sum_{i=1}^{K-1} p_i < 1, \quad 0 < \alpha_i, \quad 0 \le \beta_i < 1 \tag{20}$$

In the NM-GARCH Model, we should have Eq. 21.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} > 0, \quad n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i-\beta_i)}{(1-\beta_i)} > 0 \tag{21}$$

$$\text{and } \omega_i + \alpha_i \frac{m}{n} > 0$$

For the NM-AGARCH model, the Eq. 22 is valid.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i(\omega_i + \alpha_i \lambda_i^2)}{(1-\beta_i)} > 0, \quad n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i-\beta_i)}{(1-\beta_i)} > 0 \tag{22}$$

$$\text{and } \omega_i + \alpha_i (\frac{m}{n} + \lambda_i^2 > 0$$

and for the NM-GRJ GARCH Model, we should have Eq. 23.

$$m = \sum_{i=1}^{K} p_i \mu_i^2 + \sum_{i=1}^{K} \frac{p_i \omega_i}{(1-\beta_i)} > 0, \; n = \sum_{i=1}^{K} \frac{p_i(1-\alpha_i - 0.5\lambda - \beta_i)}{(1-\beta_i)} > 0 \tag{23}$$

$$\text{and } \omega_i + (\alpha_i + 0.5\lambda)\frac{m}{n} > 0$$

According to Alexander and Lazar (2005), both models have persistent asymmetry, when the conditional returns density is a mixture of normal density components having different means; it is generated by the difference between the expected returns under different market conditions. However, only the NM-AGARCH and NM-GRJ GARCH models have dynamic asymmetry emerging when the λ_i parameters in the component variance processes capture time-varying short-term asymmetries arising from the leverage effect. If λ_i is positive, the conditional variance is higher following a negative unexpected return at time t – 1 than following a positive unexpected return. In the markets, since negative news corresponds to a negative unexpected return, positive λ_i should be expected.

One of the assumptions in linear equation is to estimate the variance with normal distribution. The log-likelihood function of the standard normal distribution is given by Eq. 24(Peters, 2001) where T is the number of observations. In normal distribution, skewness and kurtosis take the value of (0, 3).

$$L_T = -\frac{1}{2}\sum_{T=1}^{T}\left[In(2\pi) + In(\sigma_t^2) + z_t^2\right] \tag{24}$$

Starting with Bollerslev(1987) and Hsieh(1989), Baillie and Bollerslev(1989) and Palm and Vlaar(1997) show that fat-tailed distributions like Student-t perform better to capture higher observed kurtosis. The log-likelihood function of the sdudent-t distribution is given by Eq. 25 (Saltoglu, 2003). Like normal distribution, student-t distribution is also a symmetric.

$$l^{t-dist}(\theta) = T\left\{In\Gamma(\frac{v+1}{2}) - In\Gamma(\frac{v}{2}) - \frac{1}{2}In[\pi(v-2)]\right\} \tag{25}$$

$$-\frac{1}{2}\sum_{t=1}^{T}(In(h_t) + (1+v)In(1+\frac{\varepsilon_t^2}{v-2})$$

The main drawback of these two distributions is that although student-t may account for fat tails, they are symmetric. Recently, Lambert ve Laurent (2001) applied skewed student-t distribution that is proposed by Fernandez ve Steel (1998) in Value at risk estimation (Peters,2001).

The main advantage of this density is that it considers both asymmetry and fat-tailed-ness(Saltoglu, 2003). If $\Gamma(.)$ denotes the gamma function in the log-likelihood of a standardized skewed student-t is given by Eq. 26(Peters, 2001).

$$l_{skewed-st} = T\left\{In\Gamma(\frac{\eta+1}{2}) - In\Gamma(\frac{\eta}{2}) - 0.5In[\pi(\eta-2)] + In(\frac{2}{\xi+\frac{1}{\xi}}) + In(s)\right\} \tag{26}$$

$$-0.5\sum_{t=1}^{T}\left\{In\sigma_t^2 + (1+\eta)In\left[1+\frac{(s\varepsilon+m)^2}{\eta-2}\xi^{-2I_t}\right]\right\}$$

Forecasting ability of Garch Models has been determined by squared daily returns, RMSE or absolute failure rate that is offered by Basle Committee on Banking Supervision (1996a, 1996b). The Basel backtesting is based on recording daily exceptions as comparing one year of Profit&Loss to a %99 one tail confidence 1 day value at risk with an exception whenever Profit&Loss<-value at risk. Since Basel backtesting procedure do not consider failure rate in shock positions we do not test models with this test. In order to compare asymmetric mixture Garch and other Garch models we use two widely used back testing procedures, Kupiec and Christoffersen test.

In Kupiec test, define f as the ratio of the number of observations exceeding Var(x) to the number of total observation (T) and pre-specified VaR level as α (Tang and Shieh, 2006). The statistic of Kupiec LR test is given by Eq. 27 (Kupiec, 1995). *LR* is distributed as chi-square distribution.

$$LR = 2\left\{\log\left[f^x(1-f)^{T-x}\right] - \log\left[\alpha^x(1-\alpha)^{T-x}\right]\right\} \tag{27}$$

The VaRs of α quantile for long and short trading position are computed as in Equation 28, 29 and 30 for normal, student-t and skewed student-t respectively(Tang and Shieh, 2006).

$$VAR_{long} = \hat{\mu}_t - z_\alpha\,\hat{\sigma}_t\,, \qquad VAR_{short} = \hat{\mu}_t + z_\alpha\,\hat{\sigma}_t \tag{28}$$

$$VAR_{long} = \hat{\mu}_t - st_{\alpha,v}\,\hat{\sigma}_t\,, \qquad VAR_{short} = \hat{\mu}_t + st_{\alpha,v}\,\hat{\sigma}_t \tag{29}$$

$$VAR_{long} = \hat{\mu}_t - skst_{\alpha,v,\xi}\,\hat{\sigma}_t\ , \quad VAR_{short} = \hat{\mu}_t + st_{\alpha,v,\xi}\,\hat{\sigma}_t \tag{30}$$

Where z_α, $st_{\alpha,v}$ and $skst_{\alpha,v,\xi}$ are left or right tail quantile at α % for normal, student-t and skewed student-t distributions respectively.

Christoffersen test (Christoffersen, 1998) focuses on the probability of failure rate and is based on testing whether $\Pr(r_t < v_t) = p$ after conditioning on all information available at time t (Sarma *et all*, 2001). The importance of testing conditional coverage arises with volatility clustering in financial time series.

Christoffersen test can be applied as follows (Saltoglu, 2003). Define $p^\alpha = \Pr(y_t < VaR_t(\alpha))$ to test $H_0 : p^\alpha = \alpha$ against $H_1 : p^\alpha \neq \alpha$. Consider $\{1(y_t < VaR(\alpha)\}$ which has a binomial likelihood $L(p^\alpha) = (1-p^\alpha)^{n_0}(p^\alpha)^{n_1}$.

where $n_0 = \sum_{t=R}^{T} 1(y_t > VaR_t(\alpha))$ and $n_1 = \sum_{t=R}^{T} 1(y_t < VaR_t(\alpha))$.

Under the null hypothesis, it becomes $L(\alpha) = (1-\alpha)^{n_0}\alpha^{n_1}$. Thus the likelihood ratio test statistics is in Eq. 31.

$$LR = -2In(L(\alpha))/L(\hat{p})) \xrightarrow{d} \chi(1) \tag{31}$$

We estimate VaR with α =0.01 and α =0.05 confidence interval and backtest VaR models with Kupiec in-sample and out-of-sample forecasting and Christoffersen in-sample and out-of-sample forecasting test. We chose %99 C.I. as Basel II requires %99 C.I. and %95 C.I. to compare VaR results with different C.I. level.

4. Data and Empirical Results

Data

Istanbul Stock Exchange Rate (ISE-100 Index) is from Bloomberg. Our dataset covers 2412 daily observations from 01/10/1996 to 11/07/2006. We constituted the series in log-differenced level. Figure 1 shows ISE Index in log-differenced series. By performing Augmented Dickey–Fuller (Dickey and Fuller, 1981) test we found that ISE Index is stationary at log differenced level (as Augmented Dickey-Fuller test of I(1) with 0 lags is equal to -48.2929 {<%1}). The estimation process is run using 10 years of data (1996-2005) while the remaining 5 year (252*5 days) is used for out-of-sample forecasting.

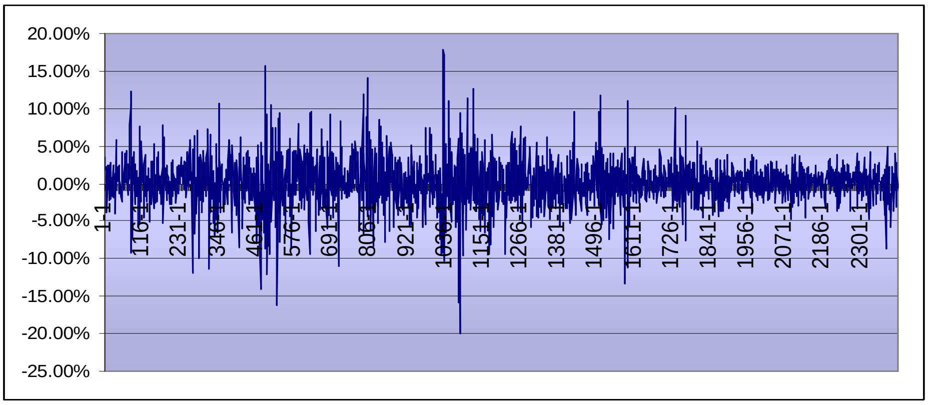

Fig. 1. ISE Log-differenced series

Empirical Results

In this subsection, we report estimation and Kupiec and Christoffersen tests results for Asymmetric Normal Mixture Garch and other Garch Models and detailed in Methodology section. We used Ox programming language (see Doornik, 1999) and parameters are estimated using Quasi Maximum Likelihood technique (Bollerslev and Woolridge, 1992) and BFGS quasi-Newton method optimization algorithm used. Estimation of Asymmetric Normal Mixture Garch is performed with modified version[‡] of Alexander and Lazar(2006) codes and other Garch models is carried out with G@rch 3.0 (Laurent and Peters, 2002).

Table 1 and Table 2 shows Garch, GRJ, FIGARCH and HYGARCH estimation results with normal, student-t and skewed student-t distributions. α and β_1 parameters for all of the models statistically significant. Student parameters (v) are statistically significant for all the Garch models and thus shows that time series is fat tailed. For the skewed student-t distribution, the asymmetric parameters (ξ) are negative and statistically significant for all Garch models. Thus show that the density distribution of ISE skewed to to left.

Table 1. Estimation Results from GARCH(1,1) and GRJ(1,1)

	Garch	**Garch-t**	**Garch-Skew**	**GJR**	**GJR-t**	**GJR-Skew**
ω	0.139**	0.169**	0.181**	0.151**	0.197**	0.204**
	(4.52)	(3.554)	(3.62)	(4.74)	(3.827)	(3.850)
α	0.110**	0.109**	0.115**	0.099**	0.088**	0.0936**
	(12.26)	(7.48)	(7.49)	(10.43)	(5.815)	(5.842)
β_1	0.880**	0.876**	0.871**	0.876**	0.867**	0.8650**
	(98.00)	(59.21)	(56.45)	(95.54)	(55.65)	(53.90)
v-Student t	-	6.560**	-	-	6.490**	-
		(7.88)			(8.032)	
ξ-Skewness	-	-	-0.056*	-	-	-0.0479*
			(-2.05)			(-1.736)
v-Skewness	-	-	6.508**	-	-	6.4409**
			(7.62)			(7.779)
$\gamma 1$-GJR	-	-	-	0.0302**	6.490**	0.0559**
				(2.50)	(8.032)	(2.468)
Volatility	0.0400219	0.0352803	0.0368966	0.0251961	0.0214036	0.0222843
LogLike	5245.17	5302.81	5304.89	5247.10	5306.9241	5308.3969
AIC	-4.375	-4.423	-4.423	-4.376	-4.425	-4.42604

[‡] The codes for Asymmetric Normal Mixture Garch in this paper are available upon request.

Table 2. Estimation Results from FIGARCH(1,d,1) and HYGARCH(1,d,1)

	Figarch Chung	Figarch Chung-t	Figarch Chung-Skew	Hygarch	Hygarch-t	Hygarch-Skew
ω	8.638**	8.346**	9.051**	1.081**	1.5037*	1.5628**
	(4.106)	(2.637)	(2.676)	(2.623)	(2.48)	(2.586)
α	0.2948**	0.2734*	0.2726**	-0.5278	-0.646*	-0.6725*
	(4.468)	(2.12)	(2.09)	(-1.42)	(-1.740)	(-1.959)
β_1	0.5216**	0.466**	0.4630**	-0.4924	-0.610	-0.6379
	(7.596)	(3.33)	(3.26)	(-1.27)	(-1.550)	(-1.746)*
v-Student t	-	6.985**	-	-	7.070**	-
		(8.00)			(7.52)	
ξ-Skewness	-	-	-0.0631*	-	-	-0.061**
			(-2.25)			(-2.19)
v-Skewness	-	-	6.988**	-	-	7.021**
			(7.714)			(7.257)
d Figarch	0.3958**	0.3679**	0.3698**	0.0403	0.0654	0.0703
	(10.66)	(6.56)	(6.64)	(0.969)	(0.982)	(1.073)
Hygarch In(α)	-	-	-	1.5988*	1.138	1.0852
				(1.71)	(1.314)	(1.383)
Volatlity	0.068605	0.0566118	0.0585214	0.007316	0.008161	0.008224
LogLike	5264.35	5316.50	5319.08	5267.16	5317.44	5319.8413
AIC	-4.390	-4.433	-4.43496	-4.39246	-4.43359	-4.43476

Estimated long memory parameter of d for Figarch and hyperbolic parameter of In(α) for HyGarch are statistically significant (Table 2).

As reported in Table 3, ω, α, β_1 and normal mixture γ(Gamma) parameter statistically significant for all of the Asymmetric Normal Mixture Models. Besides student-t and skewed student-t parameters v-Student t, ξ-Skewness and v-Skewness are also statistically significant. These results shows that Asymmetric Normal Mixture Garch models may perform better and this hypothesis can be tested with backtesting procedures such as Kupiec and Christoffersen tests.

Table 3. Estimation Results from NORMAL MIXTURE-AGARCH(1,1)

	NM-AGARCH	NM-AGARCH-t	NM-AGARCH-Skew
ω	0.155711**	0.190152 **	0.199841 **
	(4.822)	(3.661)	(3.732)
α	0.116320 **	0.119105 **	0.122624 **
	(12.26)	(7.554)	(7.542)
β_1	0.873762 **	0.863852 **	0.861064**
	(92.82)	(54.26)	(52.75)
v-Student t	-	6.469056 **	-
		(8.071)	
ξ-Skewness	-	-	-0.047852 *
			(-1.734)
v-Skewness	-	-	6.420465 **
			(7.820)
γ-Normal	0.002845 **	0.005639 **	0.005237 **
Mixture	(2.743)	(3.052)	(2.825)
Volatility	0.0408031	0.036578	0.0378326
LogLike	5247.2477	5307.6638	5309.1297
AIC	-4.37667	-4.42626	-4.42665

Table 4 shows Root Mean Squared Errors(RMSE), Mean Squared Errors(MSE), information criteria test and Nyblom test(Nyblom, 1994) results. Nyblom tests statistics shows that all of the models' parameters are stable.

Table 4. Forecast Evaluation Measures*

Method	MSE	RMSE	Akaike	$Q^2(10)$**	Nyblom test
Garch-Normal	4.16e-007	0.000645	-4.268528	17.639 [0.0241018]	1.76915
Garch-t	4.165e-007	0.0006454	-4.317390	17.7709 [0.0230116]	2.12799
Garch-Skew	4.169e-007	0.0006457	-4.317207	17.437 [0.0258668]	2.50175
GRJ-Normal	4.168e-007	0.0006456	-4.269172	15.9615 [0.042934]	2.0413
GRJ-t	4.192e-007	0.0006475	-4.320163	14.832 [0.0624966]	2.56872
GRJ-Skew	4.195e-007	0.0006477	-4.319635	14.7126 [0.0649816]	2.9462
Figarch-Normal	4.14e-007	0.0006434	-4.282631	18.8561 [0.0156486]	0.974128
Figarch-t	4.147e-007	0.000644	-4.327248	19.8371 [0.0109701]	1.04441
Figarch-Skew	4.151e-007	0.0006443	-4.327241	19.9293 [0.0106068]	1.31536
Hygarch-Normal	4.165e-007	0.0006454	-4.284598	16.6122 [0.0344101]	1.27277
Hygarch-t	4.175e-007	0.0006461	-4.327595	17.1764 [0.0283242]	1.18444
Hygarch-Skew	4.176e-007	0.0006462	-4.327535	17.1236 [0.0288478]	1.46686
NMAGARCH-N.	5.239e-007	0.0007238	-4.268447	16.9429 [0.0307096]	1.98177
NMAGARCH –t	5.240e-007	0.0007301	-4.319692	16.5072 [0.0356698]	2.62355
NMAGARCH-Skew	5.239e-007	0.0007238	-4.319203	16.3212 [0.0380071]	2.99794

** 1 day ahead out-of-sample forecasting based on 252 days evaluation.*
*** Q-Statistics on Squared Standardized Residuals*

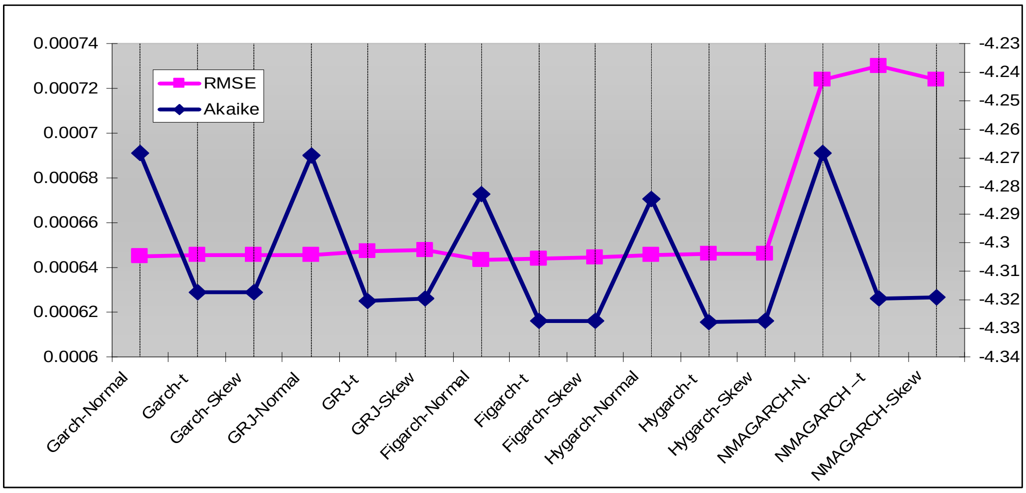

Fig. 2. RMSE and Akaike Values

As reported in Çifter (2004), RMSE or MSE may not be adequate backtesting test as these tests do not consider tail probability and overshooting effects. This can be seen in Figure 2 as RMSE is maximum for NMGARCH models where Akaike criteria tests are not maximum for NMGARCH models. Kupiec and Christoffersen tests can be more consistent to compare Garch models.

We compared VaR models with Kupiec test for long and short trading positions. We define a failure rate for long trading position as percentage of negative returns smaller than one-step ahead VaR for long position (left tail of the density distribution of the returns) and a failure rate as the percentage of positive returns larger than one-step ahead VaR for short position (right tail of the density distribution of the returns).

The empirical results based on Kupiec in-sample forecasting test are summarized in Table 5 and Figure 3. The table contains Kupiec failure rates for short and long position VaR. Number of in-sample-forecasting is 15 days and confidence interval(C.I.) is chosen as with α=0.01 and α=0.05. Table 5 can be read as follow. If the model is estimated accurately, it should explain the actual observations very well. The failure rate should be equal to the pre-specified VaR level, and Kupiec LR test would not reject its null hypothesis as failure rate equals to α (Tang and Shieh, 2006).

In sample VaR results for long and short trading positions are reported in Table 5. The empirical results of Kupiec in-sample forecasting test shows that NMAGARCH with Gaussian distribution for short position and Figarch(1,d,1) with skewed student-t distribution performs better for α=0.05 where NMAGARCH with student-t for short position and GRJ with student-t and Hygarch with skewed student-t distribution for long position performs better for α=0.01. These results show that none of the model outperforms other models based on Kupiec in-sample forecasting.

Since in-sample forecasting estimates VaR with only know the past performance, out-of-sample forecasting is more consistent. Our out-of-sample forecast evaluation uses one step ahead prediction for 252×5 days forecast sample. Out of sample VaR results for long and short trading positions are reported in Table 6 and Figure 4. The empirical results of Kupiec out-of-sample forecasting test shows that Figarch(1,d,1) with skewed student-t distribution and Hygarch(1,d,1) with skewed student-t distribution for short position and Hygarch(1,d,1) with student-t distribution performs better for α=0.05 where NMAGARCH with Gaussian distribution for short position and GRJ with Gaussian distribution and NMAGARCH with Gaussian distribution for long position performs better for α=0.01. The empirical evidence is in favor of the Figarch with skewed student-t, Hygarch with skewed student-t, GRJ with Gaussian, GRJ with student-t and *NMAGARCH* with Gaussian distribution based on Kupiec in-sample and out-of-sample forecasting.

Christoffersen test VaR results for in-sample and out-of-sample forecasting are reported in Table 7 and Figure 5. The empirical results in-of-sample forecasting results shows that NM-AGARCH with Gaussian distribution for α=0.05 and FIGARCH(1,d,1) with Gaussian distribution for α=0.01 perfoms better where out-of-sample forecasting results show that GARCH(1,1) with Gaussian distribution for α=0.05 and NMAGARCH with student-t for α=0.01 performs better.

Empirical results based on Kupiec and Christoffersen tests show that volatility model should be chosen in accordance with confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord requires accurate volatility model, which is statistically significant at 99 % confidence level.

Figure 6 shows out-of-sample estimation for GARCH and NM-AGARCH with gaussian and skewed student-t distribution. NM-AGARCH captures fat-tailed behavior of the data(shocks) better than GARCH.

Table 5. In-Sample-Forecasting Kupiec Test

In Sample Forecasting %95 Confidence Interval$^{\delta}$						
	VaR for Short position			**VaR for Long position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
Garch-Normal	0.94894	0.02651	0.87066	0.043134	1.1801	0.27734
Garch-t	0.94366	0.92453	0.33629	0.046655	0.27346	0.60102
Garch-Skew	0.94190	1.4943	0.22155	0.044014	0.89142	0.34509
GRJ-Normal	0.94630	0.31955	0.57188	0.039613	2.7699	0.09605**
GRJ-t	0.94190	1.4943	0.22155	0.040493	2.3054	0.12893*
GRJ-Skew	0.94014	2.1927	0.13866*	0.038732	3.2807	0.07009**
Figarch-Normal	0.94718	0.18649	0.66586	0.046655	0.27346	0.60102
Figarch-t	0.93662	3.9618	0.04654**	0.047535	0.14761	0.70083
FigarchSkew	0.93926	2.5891	0.10760*	0.048415	0.060655	**0.80546**
Hygarch-Normal	0.94718	0.18649	0.66586	0.046655	0.27346	0.60102
Hygarch-t	0.94454	0.68909	0.4064	0.047535	0.14761	0.70083
Hygarch-Skew	0.94102	1.8277	0.17640	0.045775	0.43889	0.50766
NMAGARCH-No.	0.95033	0.0056353	**0.94016**	0.037980	7.9194	0.004890*
NMAGARCH –t	0.94783	0.23440	0.62828	0.041319	4.0301	0.04469**
NMAGARCH-Skew	0.94491	1.2678	0.26019	0.038815	6.8146	0.00904**
In Sample Forecasting %99 Confidence Interval$^{\delta}$						
	VaR for Short position			**VaR for Long position**		
	Failure Rate	**Kupiec LR**	**p-value**	**Failure Rate**	**Kupiec LR**	**p-value**
Garch-Normal	0.98680	1.0703	0.30087	0.017606	5.4119	0.02000**
Garch-t	0.99032	0.011646	0.91406	0.010563	0.035762	0.85001
Garch-Skew	0.98856	0.22852	0.63263	0.010563	0.035762	0.85001
GRJ-Normal	0.98504	2.4542	0.11721*	0.018486	6.6087	0.01014**
GRJ-t	0.99120	0.17138	0.67889	0.0096831	0.011646	**0.91406**
GRJ-Skew	0.99120	0.17138	0.67889	0.0079225	0.53322	0.46526
Figarch-Normal	0.98327	4.3170	0.03773**	0.018486	6.6087	0.01014**
Figarch-t	0.98944	0.035762	0.85001	0.010563	0.035762	0.85001
Figarch-Skew	0.98856	0.22852	0.63263	0.011444	0.22852	0.63263
Hygarch-Normal	0.98327	4.3170	0.03773**	0.017606	5.4119	0.02000**
Hygarch-t	0.99120	0.17138	0.67889	0.011444	0.22852	0.63263
Hygarch-Skew	0.98944	0.035762	0.85001	0.0096831	0.011646	**0.91406**
***NMAGARCH*-No.**	0.98539	4.4988	0.03391**	0.013356	2.4657	0.11636
***NMAGARCH* -t**	0.98998	6.7415e-5	**0.99345**	0.0095993	0.039377	0.84270
***NMAGARCH*-Skew**	0.98790	1.0035	0.31647	0.0087646	0.38543	0.53471

**, ** are %5 and %10 confidence level respectively.*

$^{\delta}$ *Number of forecast:15 days*

Table 6. Out-of-Sample Forecasting Kupiec Test

Out-of-Sample Forecasting %95 Confidence Interval[δ]	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
Garch-Normal	0.97222	15.505	8e-005**	0.025397	19.442	1e-005**
Garch-t	0.97222	15.505	8e-005**	0.027778	15.505	8e-005**
Garch-Skew	0.96905	11.071	0.00087**	0.026190	18.068	2e-005**
GRJ-Normal	0.97143	14.312	0.00015**	0.026190	18.068	2e-005**
GRJ-t	0.96905	11.071	0.00087**	0.028571	14.312	0.00015**
GRJ-Skew	0.96905	11.071	0.00087**	0.027778	15.505	8e-005**
Figarch-Normal	0.96825	10.099	0.00148**	0.029365	13.177	0.00028**
Figarch-t	0.96429	5.9868	0.014413*	0.033333	8.3072	0.00394**
Figarch-Skew	0.95873	2.1441	**0.14312**	0.032540	9.1778	0.00244**
Hygarch-Normal	0.96508	6.7128	0.009572*	0.031746	10.099	0.00148**
Hygarch-t	0.96032	3.0295	0.081763*	0.035714	5.9868	**0.014413***
Hygarch-Skew	0.95873	2.1441	**0.14312**	0.030159	12.097	0.00050**
***NMAGARCH*-No.**	0.97222	15.505	8.22-e5**	0.027778	15.505	8.22-e5**
***NMAGARCH* -t**	0.97063	13.177	0.00026**	0.027778	15.505	8.22e-5**
***NMAGARCH*-Skew**	0.96984	12.097	0.0005 **	0.028571	14.312	0.00015**

Out-of-Sample Forecasting %99 Confidence Interval[δ]	VaR for Short position			VaR for Long position		
	Failure Rate	Kupiec LR	p-value	Failure Rate	Kupiec LR	p-value
Garch-Normal	0.99365	1.9489	0.16271	0.007936	0.58318	0.44507
Garch-t	0.99683	8.0799	0.00447**	0.0055556	2.9961	0.083466*
Garch-Skew	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
GRJ-Normal	0.99444	2.9961	0.08346**	0.0079365	0.0079365	**0.44507**
GRJ-t	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
GRJ-Skew	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
Figarch-Normal	0.99444	2.9961	0.083466*	0.0095238	0.029325	0.86403
Figarch-t	0.99524	4.3316	0.037411*	0.0087302	0.21442	0.64333
Figarch-Skew	0.99444	2.9961	0.083466*	0.0071429	1.1539	0.28274
Hygarch-Normal	0.99365	1.9489	0.16271	0.011111	0.15167	0.69695
Hygarch-t	0.99444	2.9961	0.083466*	0.0063492	1.9489	0.16271
Hygarch-Skew	0.99444	2.9961	0.083466*	0.0063492	1.9489	0.16271
***NMAGARCH*-No.**	0.99286	1.1539	**0.28274**	0.0079365	0.58318	**0.44507**
***NMAGARCH*-t**	0.99603	6.0036	0.01427**	0.0055556	2.9961	0.083466*
***NMAGARCH*-Skew**	0.99603	6.0036	0.0142 **	0.0047619	4.3316	0.03741 *

**, ** are %5 and %10 confidence level respectively.*

*[δ] Number of forecast:252*5 days and 1 day ahead*

Table 7. Christoffersen Test

In Sample Forecasting %95.00 Confidence Interval [&]

Method	LR	p-value	Method	LR	p-value
Garch-Normal	5.7993	0.016032	**Figarch-Skew**	2.0046	0.72034
Garch-t	4.0301	0.044694	**Hygarch-Normal**	3.6410	0.056372
Garch-Skew	1.4909	0.22207	**Hygarch-t**	1.2635	0.26099
GRJ-Normal	7.9194	0.0048908	**Hygarch-Skew**	0.00563	0.94016
GRJ-t	4.0301	0.044694	***NMAGARCH-No.***	7.98953	**0.003214**
GRJ-Skew	2.2914	0.13009	***NMAGARCH-t***	4.157480	0.039824
Figarch-Normal	5.7993	0.016032	***NMAGARCH-Skew***	5.93530	0.087203
Figarch-t	2.0047	0.15681			

In Sample Forecasting %99.00 Confidence Interval [&]

Method	LR	p-value	Method	LR	p-value
Garch-Normal	2.4657	0.11636	**Figarch-Skew**	1.4241	0.23273
Garch-t	0.37426	0.54069	**Hygarch-Normal**	1.4241	**0.021389**
Garch-Skew	1.4241	0.23273	**Hygarch-t**	0.65273	0.41914
GRJ-Normal	2.4657	0.11636	**Hygarch-Skew**	1.9122	0.16672
GRJ-t	6.7415e-5	0.99345	***NMAGARCH-No.***	2.4952	0.09250
GRJ-Skew	0.37426	0.54069	***NMAGARCH-t***	0.039377	0.84272
Figarch-Normal	6.1472	0.013162	***NMAGARCH-Skew***	0.170710	0.67948
Figarch-t	0.17071	0.67948			

Out-of-Sample Forecasting %95.00 Confidence Interval [δ]

Method	LR	p-value	Method	LR	p-value
Garch-Normal	19.442	**1.0368e-005**	**Figarch-Skew**	3.0295	0.081763
Garch-t	15.505	8.2294e-005	**Hygarch-Normal**	10.099	0.0014837
Garch-Skew	12.097	0.00050507	**Hygarch-t**	5.9865	0.014413
GRJ-Normal	18.068	2.1312e-005	**Hygarch-Skew**	4.0816	0.043354
GRJ-t	14.312	0.00015486	***NMAGARCH-No.***	15.505	8.2294e-005
GRJ-Skew	14.312	0.00015486	***NMAGARCH-t***	15.507	8.2296e-005
Figarch-Normal	13.177	0.00028346	***NMAGARCH-Skew***	14.312	0.00015489
Figarch-t	8.3072	0.0039488			

Out-of-Sample Forecasting %99.00 Confidence Interval [δ]

Method	LR	p-value	Method	LR	p-value
Garch-Normal	0.58318	0.4451	**Figarch-Skew**	0.21442	0.64333
Garch-t	2.9961	0.083466	**Hygarch-Normal**	0.15167	0.69695
Garch-Skew	1.1539	0.28274	**Hygarch-t**	1.9489	0.16271
GRJ-Normal	0.58318	0.44507	**Hygarch-Skew**	0.21442	0.64333
GRJ-t	2.9961	0.083466	***NMAGARCH-No.***	0.68250	0.23510
GRJ-Skew	2.9961	0.083466	***NMAGARCH-t***	3.12450	**0.051542**
Figarch-Normal	0.029325	0.86403	***NMAGARCH-Skew***	2.98457	0.085287
Figarch-t	0.21442	0.64333			

[&] *Number of forecast(in-sample):15 days ahead*

[δ] *Number of forecast(out-of-sample): 1 day ahead for 252*5 days sample*

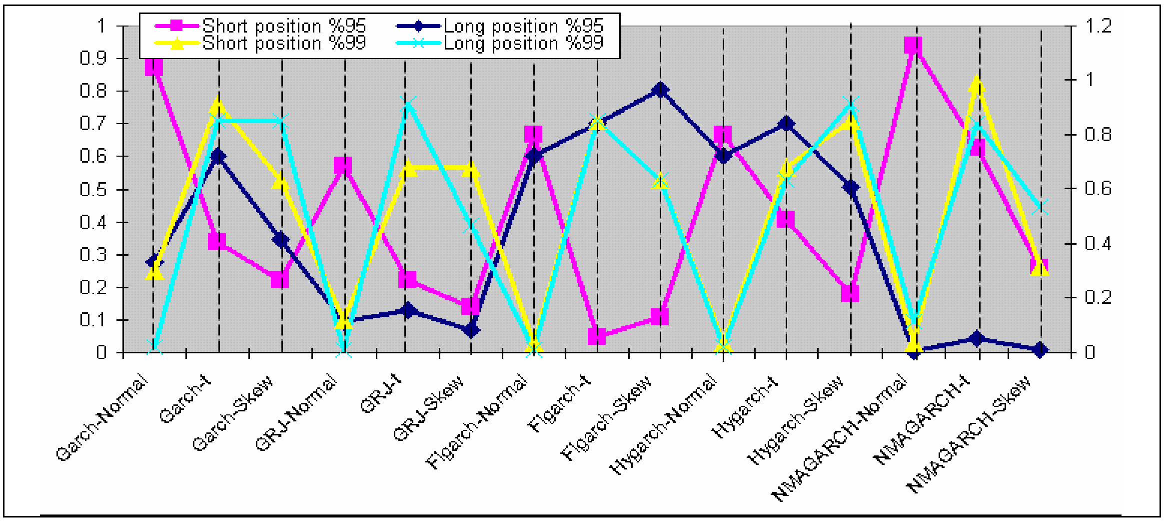

Fig. 3. In-sample Kupiec test p-value

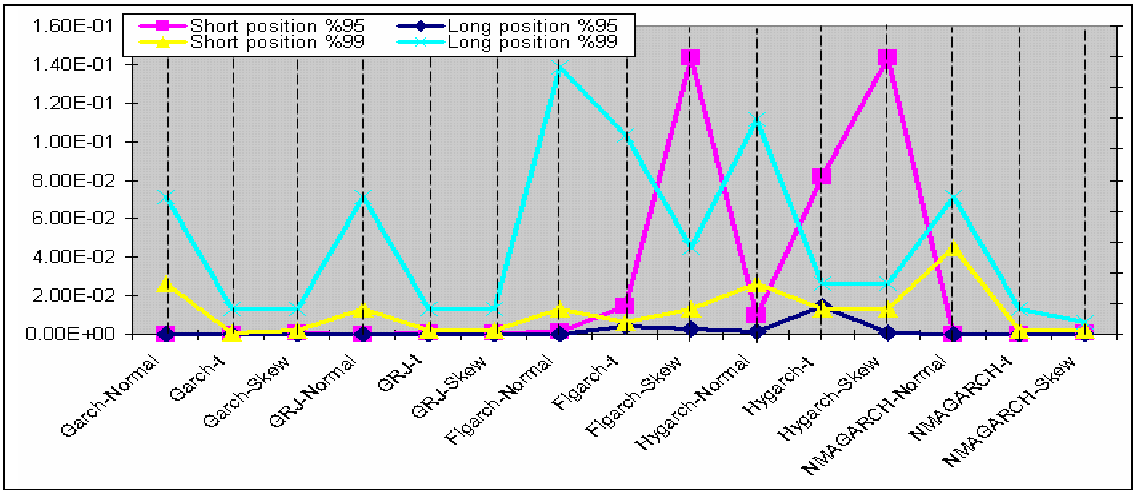

Fig. 4. Out-of-sample Kupiec test p-value

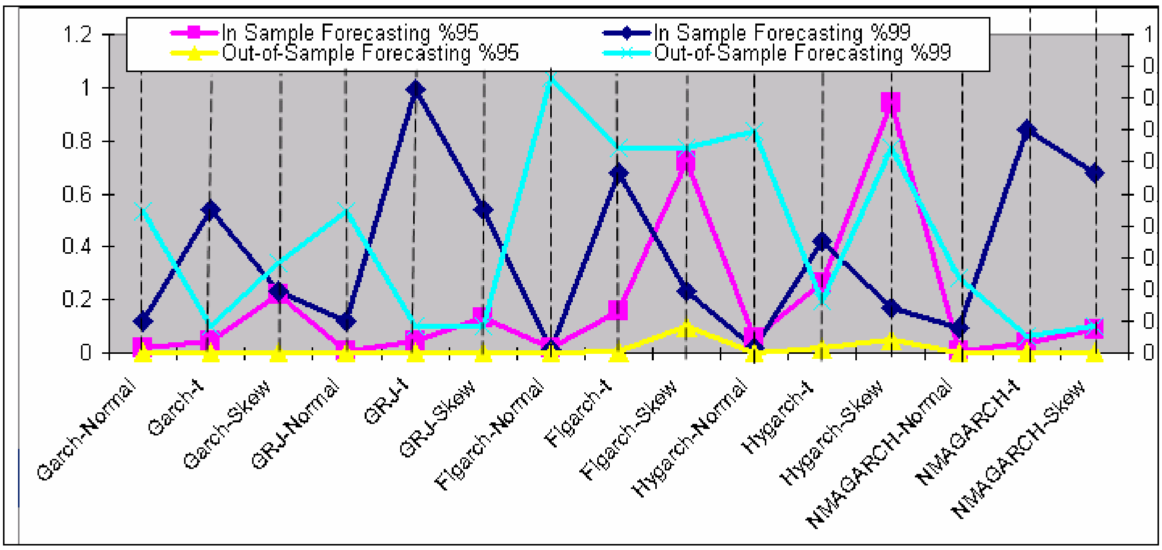

Fig. 5. Christoffersen Test p-value

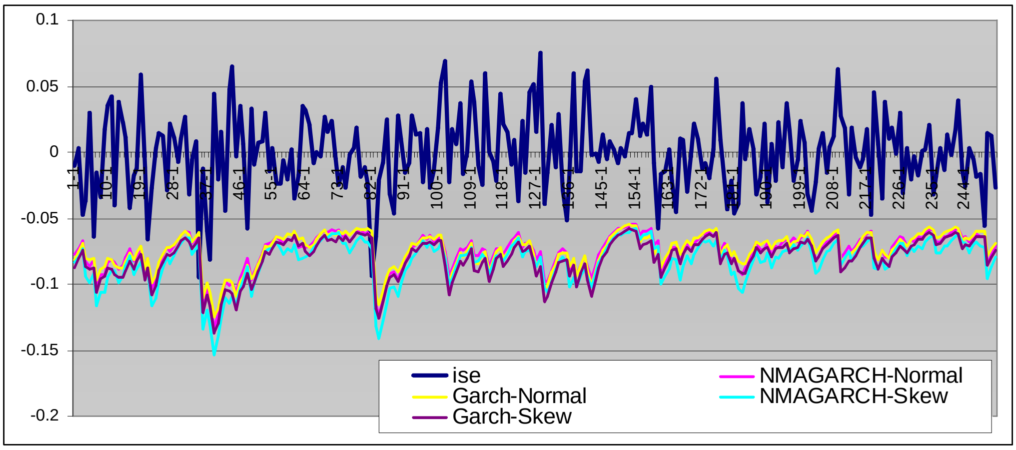

Fig. 6. Out-of-Sample Forecasting (Last 252 days)

5. Conclusion

Though volatility in stock returns provides opportunity in earning profit for traders, it is a threat for risk managers in balancing risk-return relationship. In emerging markets, return volatility is relatively high due to low market volume, unstable political and economic conditions, and hot money from international investment portfolios. High volatility and non-linear returns in stock prices require advanced volatility measurement models based on non-normal distribution of returns. They should catch the fat tails and regime switches, which are not easy to be estimated and modeled with static econometric models.

In this paper, the return volatility of stocks traded in the Istanbul Stock Exchange is estimated by different GARCH models. The research is especially interested in the predictive performance of Asymmetric Normal Mixture Garch (NMAGARCH) based on Kupiec and Christoffersen tests for the Istanbul Stock Exchange National 100 Index. In this respect, this article includes the first research employing the NMAGARCH model in Turkish equity markets. What is more, it has original contribution to the finance literature by conducting reality check of the NMAGARCH model with comparing the classical GARCH models.

By examining fifteen GARCH models with alternative return distribution assumptions, the paper shows that the NMAGARCH perform better based on 99 %confidence interval out-of-sample forecasting Christoffersen test. On the other hand, Figarch with skewed student-t, Hygarch with skewed student-t, GRJ with normal, GRJ with student-t and *NMAGARCH* with Gaussian distribution perform better based on 95 % confidence interval out-of-sample forecasting Christoffersen test and Kupiec tests.

The empirical evidence has a crucial concluding remark in prediction of stock market volatility. The results show that volatility model should be chosen in accordance with confidence interval and trading positions. However, NMAGARCH model has better predictive performance for higher confidence interval. The Basel II Accord requires accurate volatility model, which is statistically significant at 99 % confidence level. The paper show that for accurate internal volatility models being proper for the Basel II Accord, advanced models based on financial computing should be constructed by examining the nature of the markets under investigation.

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